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Smoothing and Interpolation in LFM-II Output

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

Smoothing

Joe Gerrity and Jack Newell found that by adding the so-called G-smoother on the output grid (Δ = 190.5 km at 60N) they reduced the noise in a very noisy 500 mb vorticity field. In eyeballing the charts, with and without the final G-smoother, I estimate about 2/3 of the noise is removed. The G-smoother in one dimension can be broken down into three elements, $\nu = \frac{1}{2}, \frac{1}{2}, -1$. Here ν is Shuman's (1957) smoothing index and is twice the weight applied to the outer two points of a 3-point smoothing operator.

In the currently operational LFM-II, the G-smoother is used on the LFM-II grid ($\Delta' = 2\Delta/3$ = 127 km at 60N), and so also is SMOHEX, which I will here call H-smoother. (H is for Hinkelmann, who was the first to use it in NWP, I believe.) The H-smoother is a 2-element operator with indices, $\nu = \frac{1}{2}, -\frac{1}{2}$.

Now, Figure 1 shows the response of the G-smoother when applied on the Δ-grid (G) and on the Δ'-grid (G$'$). Figure 1 also shows the response of the H-smoother on the two grids (H and H$'$). The abscissa is $\cos r\Delta$, but note that the region $-1 \leq \cos r\Delta \leq 0$ is folded about -1 so that it is repeated in the left 1/3 of the figure. This is to account for those parts of the spectrum in Δ'-grid which the NMC graphics system aliases in the Δ-grid.

The response is on a component

$$f = \exp irx$$

where x is distance on a map. The result of passing an element on f is

$$\overline{f} = f \ (1 - \nu \ \xi)$$

$$\xi = 1 - \cos r\Delta$$

If we let

$$rx = \frac{2\pi kj}{J}$$

$$j = x/\Delta$$

where J is the number of grid intervals in the given region of x, then J/k is the wave-length measured in units of Δ. Similarly, J'/k is the wave-length measured in units of Δ'. Note that waves between $2\Delta'$ and $3\Delta'$ long are aliased by the NMC graphics system into waves between 2Δ and 4Δ long.

In Figure 2, the curve marked H'G' shows the response of the currently operational smoothers in LFM-II. Note that aliased components are effectively removed.

The curve marked HG shows the response of the smoothers that were operational in LFM-I. The response HG must be regarded as acceptable in the region $3 \leq J'/k$, because of the quality of LFM-I forecast output.

The curve marked H'G'G shows the response of all output smoothers in Gerrity's and Newell's experiment. Note that the responses HG and H'G'G are nearly the same except in the region, $J'/k < 3$.

The curve marked G'G shows the response of passing the G-smoother in both the Δ- and Δ'-grids. The conclusion is that the H' operator did not contribute to Gerrity's and Newell's good result.

In the operational LFM-II, therefore, the H'-smoother should be removed from any fields on which the G'-smoother is used. On those fields, the G-smoother should be substituted for the H'-smoother.

Interpolation

Now, I come to the question of interpolation. Does interpolation from the Δ'-grid to the Δ-grid degrade the output of LFM-II? The answer is yes, to some extent, and interpolation procedures for this purpose should be improved.

Again, consider

$$f = \exp irx \tag{1}$$

$$x = j\Delta = j'\Delta'$$

$$\Delta' = 2\Delta/3$$

$$rx = \frac{2\pi kj}{J}$$

I will consider $r\Delta'$ only in the range

$$-\pi \leq r\Delta' \leq \pi$$

Then

$$-3\pi/2 \leq r\Delta \leq 3\pi/2$$

The pattern of points in the Δ- and Δ'-grid is shown in figure 3.

j': 0 1 2 3 4 5 6

Δ'

Δ

j : 0 1 2 3 4

FIGURE 3.

As can be seen in Figure 3, only points at odd j need be interpolated. Short of a Fourier analysis, such interpolation must of necessity consist of the 2-point smoother in the Δ'-grid (j'-pattern: $\frac{1}{2}$ $\frac{1}{2}$) and possibly one or more three-point elements (Shuman, 1957). The response (R_o) then, is

$$R_o = \cos \tfrac{1}{2} r\Delta' \, [1-(\upsilon_o)_1 \rho'][1-(\upsilon_o)_2 \rho'] \cdots$$

$$\rho' = 1-\cos r\Delta'$$

but only at odd j. The interpolated value at odd j, then, is

$$\bar{f}_o = f \, R_o \tag{2}$$

Now, at even j, I will consider not only $\bar{f} = f$, which is a possible value of f to be used, but also

$$\bar{f}_e = f \, R_e \tag{3}$$

where R_e is the response of a smoothing operation on f in the Δ'-grid:

$$R_e = [1-(\upsilon_e)_1 \rho'][1-(\upsilon_e)_2 \rho'] \cdots$$

but only at even j.

With (2) and (3) I assert for the output Δ-grid, that

$$\bar{f} - f = \tfrac{1}{2} f (1 - \cos \pi j)(R_o - 1) + \tfrac{1}{2} f (1 + \cos \pi j)(R_e - 1)$$

$$= f \left[\tfrac{1}{2}(R_o + R_e) - 1\right] + \tfrac{1}{2} f \cos \pi j \,(R_e - R_o)$$

or,

$$\bar{f} = \tfrac{1}{2} f (R_e + R_o) + \tfrac{1}{2} f \cos \pi j \,(R_e - R_o)$$

Now, $f \cos \pi j$ is a new component, not contained in (1). Thus, the interpolation involves interactions in the spectrum.

However, the interactions in the spectrum are limited. For instance, let

$$s\Delta = r\Delta \pm \pi$$

I use the upper sign if $r\Delta$ is negative, the lower if $r\Delta$ is positive, so that

$$-\pi \le s\Delta \le +\pi$$

I also define ℓ, S, σ, corresponding to k, R, ρ:

$$sx = \frac{2\pi\ell j}{J}$$

$$S_o = \cos \tfrac{1}{2} s\Delta' \, [1-(\nu_o)_1 \, \sigma'][1-(\nu_o)_2 \, \sigma'] \cdots$$

$$S_e = [1-(\nu_e)_1 \, \sigma'][1-(\nu_e)_2 \, \sigma'] \cdots$$

$$\sigma' = 1 - \cos s\Delta'$$

Now, I define g:

$$g = \exp isx$$

and note that $\cos \pi j = \exp(\pm i\pi j)$ at integer j, and consider the new component $f \cos \pi j$ in the interpolated function $\bar{f}$ in Δ-grid:

$$f \cos \pi j = \exp i(rx \pm \pi j)$$

But, since $x = j\Delta$,

$$f \cos \pi j = \exp isx = g$$

Thus, in the interpolation, f and g interact with each other but, importantly, with no other components in the spectrum. The two together, then, are a complete system for my purpose here.

The present operational interpolation operator is simply the 2-point smoother in Δ'-grid.

Thus $\nu_o = 0$ and

$$R_o = \cos \tfrac{1}{2} r\Delta'$$

$$S_o = \cos \tfrac{1}{2} s\Delta'$$

(R_o acts on f, S_o on g.) Points at even j are simply read unchanged out of the Δ'-grid. Thus, $\nu_e = 0$ and

$$R_e = S_e = 1$$

In Figure 4, the two curves marked "$\nu_e = \nu_o = 0$" show the appropriate responses of the present interpolation system. The upper curve is $\frac{1}{2}(R_e + R_o)$, the lower $\frac{1}{2}(S_e - S_o)$.

The meaning of the lower curve is that the ordinate represents the fraction of the $s(\ell)$-component which is added to the $r(k)$-component. J/ℓ in the figure shows the wave-length of the interacting s-component. Note, by the way, that $\cos s\Delta = - \cos r\Delta$ and that $J/\ell = - (J/k)/(\frac{1}{2}J/k-1)$.

Figure 4 shows that 25% of the 2Δ-long component ($J/\ell = -2$) adds to the mean ($J/k = \infty$), and that all long waves are significantly disturbed by short ones in the interpolation. This is not a good feature, but the problem is somewhat alleviated by the H'G'-smoother (Figure 2).

I have calculated like responses for a number of interpolation systems, and have developed one that is superior to the operational system. The interpolation in the new system at odd points is cubic, for which only a single element is required, with $\nu_o = -\frac{1}{4}$. The points at even j are smoothed-desmoothed, requiring two elements, with the two smoothing indices (one the negative of the other) chosen so that the mean is unaffected. The responses for a smoother-desmoother are

$$R_e = 1 - \nu_e^2 \rho'^2$$

$$S_e = 1 - \nu_e^2 \sigma'^2$$

I have derived

$$\nu_e^2 = 5/36$$

$$\nu_e = \pm\sqrt{5}/6$$

In Figure 4, the curves marked, "$\nu_e^2 = 5/36$, $\nu_o = -\frac{1}{4}$," show the appropriate responses for the new system. Note that both the upper curve and the lower curve show great improvement over the operational system. The reduction of amplitudes for the long waves is reduced by at least an order of magnitude, and so are the interactions in the spectrum.

The conclusion is that, although the present interpolation may not be leading to serious problems, it should be modified. At even j, the "interpolated" value should be the result of a 5-point operator (25-point in two dimensions) with $\nu_e = \pm\sqrt{5}/6$. At odd j, the interpolation should be cubic, i.e., $\nu_o = -\frac{1}{4}$.

Reference

Shuman, F. G., 1957: Numerical methods in weather prediction: II. Smoothing and filtering. Monthly Weather Review, 85, 357-361.

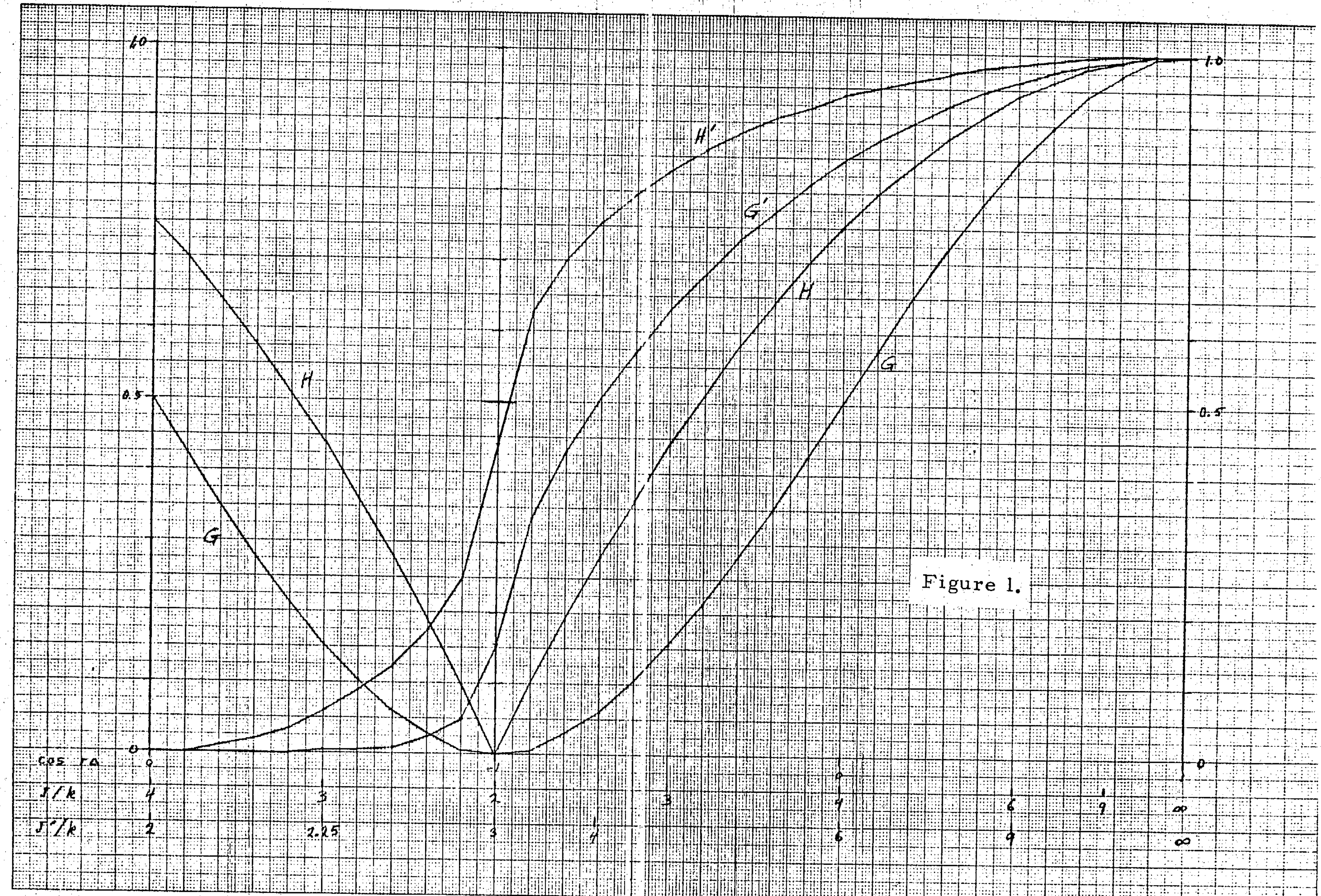


Figure 1.

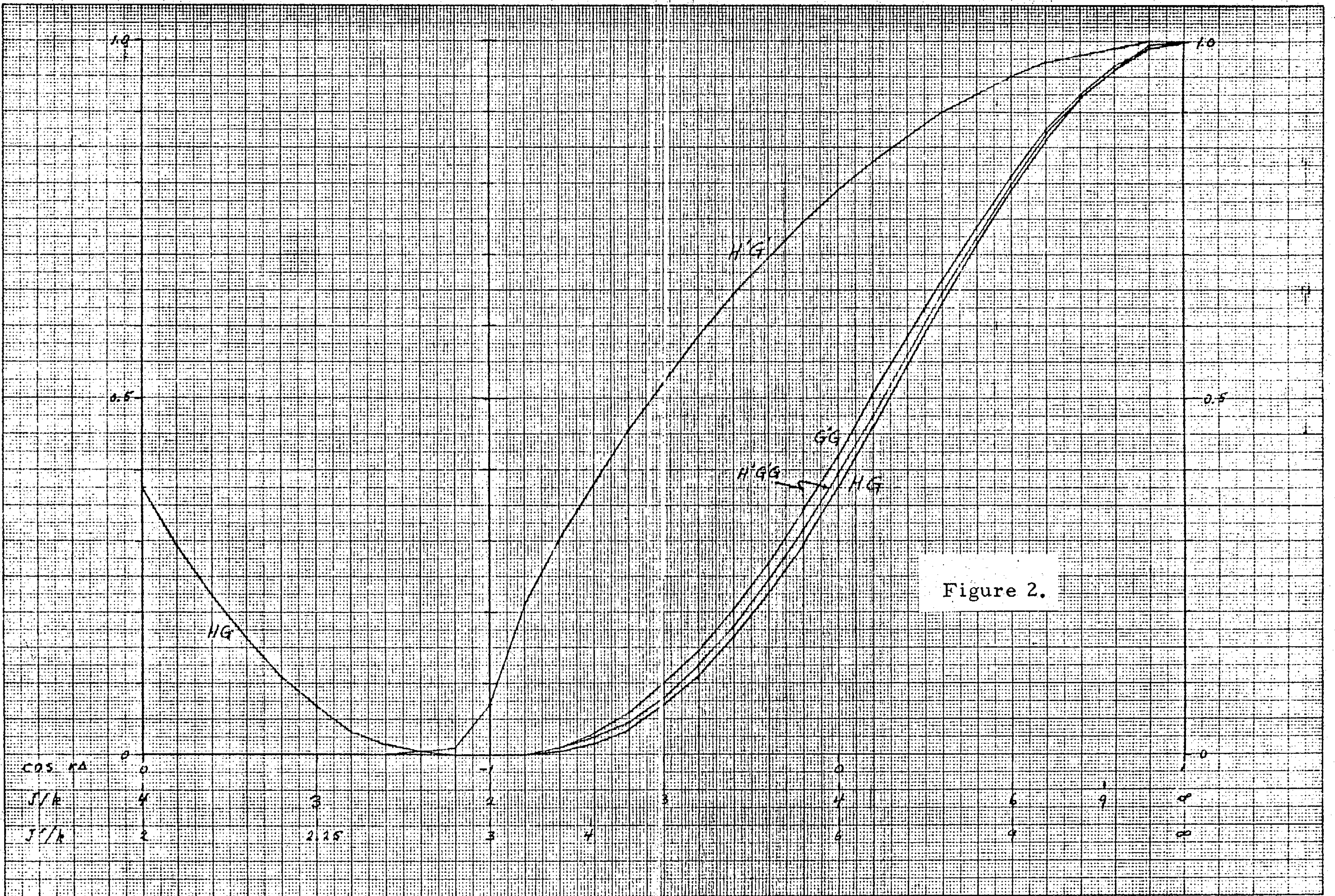
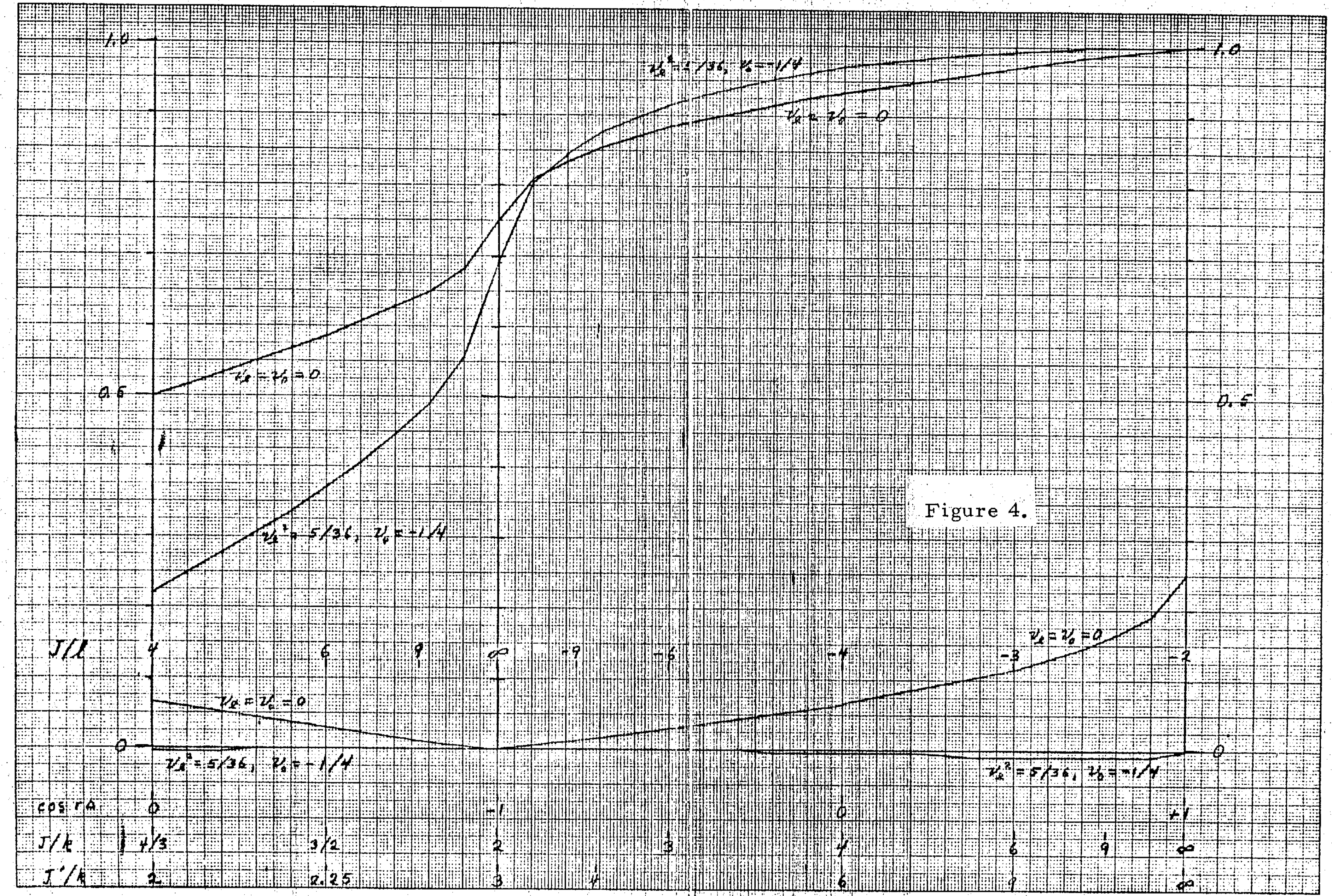


Figure 2.



1.0
0.6
0
0.5
$\nu_k^2 = 5/36,\ \nu_0 = -1/4$
$\nu_k = \nu_0 = 0$
J/l
4
6
9
∞
-9
-6
-4
-3
-2
cos rA
0
-1
0
+1
J/k
4/3
3/2
2
3
4
6
9
∞
J'/k
2
2.25
3
4
6
9
∞

Figure 4.